RRB-JE 2024

Railway Recruitment Board

Junior Engineer Examination

Mechanical Engineering

Strength of Materials

Well Illustrated **Theory** *with*
Solved Examples and **Practice Questions**

MADE EASY PUBLICATIONS

Strength of Materials

Contents

Chapter 01

Properties of Metals, Simple Stress-Strain and Elastic Constants

1.1 Introduction

- Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.
- So the objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials so these quantities are found for all values of load upto the failure load, then we will have a complete picture of the mechanical behaviour of the body.

1.2 Normal Stress

- It is the internal resistance offered by the body against external loading or deformation which is force per unit area.
- Stress induced in the material depends upon the nature of force, point of application and cross-section area of material. Stress can be tensile or compressive in nature depending on the nature of load.

$$\sigma = \frac{P}{A} \text{ N/mm}^2 \text{ or MPa}$$

- Sign convention: Tensile stress = +ve
  Compressive stresses = –ve
- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansions allowed then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.
- On the basis of cross-section area considered during calculation of stresses. Direct stresses can be of following two types.
  1. Engineering stresses or nominal stresses.
  2. True stresses or actual stresses

Mathematically:

Engg. stress, $\sigma = \frac{P}{A_0}$

A_0 = Original cross-section area of specimen taken

True stress, $\sigma_t = \frac{P}{A_a}$

A_a = Actual cross-section area of specimen at any time of loading i.e., changed area of cross-section due to loading.

$$A_a = A_0 + \Delta A$$

- In tension, $A_a < A_0$ $(\because L_a > L_0$ (as $L_a = L_0 + \Delta L$) and $A_a L_a = A_0 L_0)$

  $\therefore$ $\sigma_t > \sigma$
- In compression, $A_a > A_0$

  $\therefore$ $\sigma_t < \sigma$

1.3 Strain

- An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. Thus, the elongation or shortening in axially loaded member per unit length is known as strain.

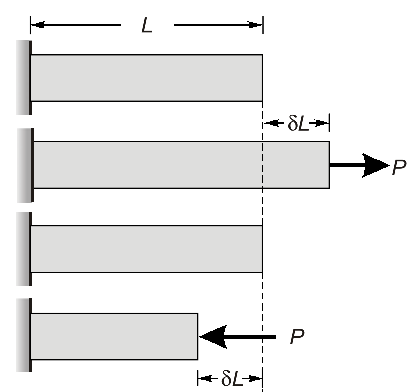


Figure 1.1

Mathematically:

$$\in = \frac{\delta L}{L}$$

- Strain is dimensionless quantity.
- It member is in tension, strain is called tensile strain.
- If member is in compression, then strain is called compressive strain.
- On the basis of length of member used in calculation of strain, stain can be of following two types.
  - Engineering or nominal strain
  - True or actual strain

1. **Engineering or nominal strain**

$$\in = \frac{\delta L}{L_0}$$

L_0 = Original length of member

2. **True or actual strain**

$$\in_T = \frac{\Delta L}{L_a}$$

L_a = Actual length of member

$$L_a = L_0 + \Delta L$$

$\oplus$ = Tension, $\ominus$ = compression.

1.4 Stress-strain Curve

1.4.1 Simple Tension Test for Mild Steel

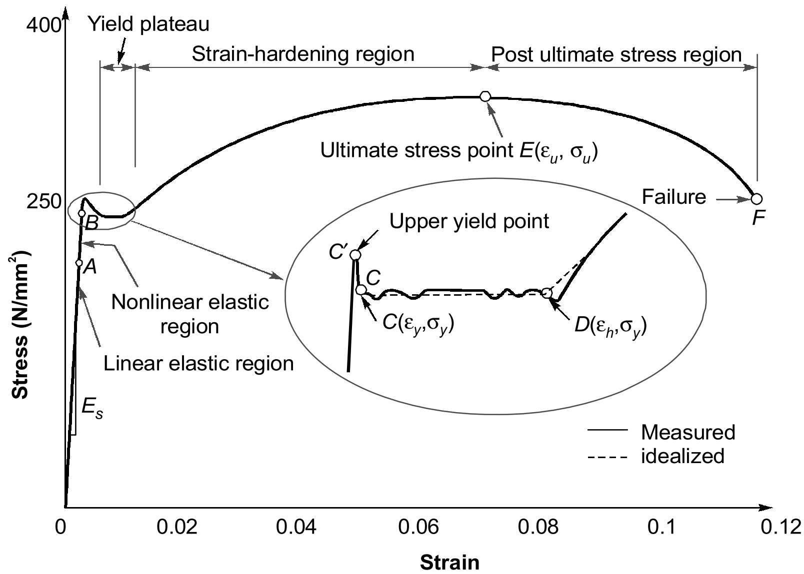


Figure 1.2

- ***A* is Limit of proportionality:** Beyond *A* linear variation ceases. Hook's law is valid in *OA*.
- ***B* is Elastic limit:** The maximum stress up to which a specimen regains its original length on removal of applied load. For mild steel *B* is very near to *A*. However, for other materials *B* may be greater than *A*.
- ***C′* is Upper yield point:** The magnitude of the stress corresponding to *C′* depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- ***C* is Lower yield point:** The stress at *C* is the yield stress (σ_y) with a typical value of σ_y = 250 N/mm^2 for mild steel. The yielding begins at this stress.
- ***CD* represents perfectly plastic region:** It is the strain which occurs after the yield point *C*, without any increase in stress. The strain corresponding to point *D* is about 1.4% and corresponding to *C* is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- ***DE* represents strain hardening:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its atomic and crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- ***E* is Ultimate point:** The stress corresponding to this point is ultimate stress (σ_y) and the corresponding strain is about 20% for mild steel.
- ***F* is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- Region between *E* and *F* is the necking region, in which area of cross-section is drastically decreased.

Do you know? Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.

Example 1.1 A rod of dimension 20 mm × 20 mm is carrying an axial tensile load of 10 kN. The tensile stress developed is

(a) 0.025 MPa (b) 0.25 MPa (c) 25 MPa (d) 250 MPa

[SSC JE : 2018]

***Solution:* (c)**

$$\sigma = \frac{P}{A} = \frac{10 \times 1000}{20 \times 20} = 25 \text{ MPa}$$

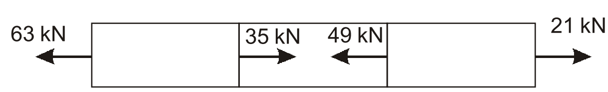
2. Ideal curve for tension is shown in the figure. However, actual behaviour is different and indicates apparently reduced yield stress in compression for mild steel. The divergence between tension and compression results is explained by Bauchinger and is called **Bauchinger effect**.
3. The stress defined as the ratio of load to original area (A_o) is known as 'engineering stress' or 'stress' or 'nominal stress'.

$$\text{Engineering stress or Nominal stress} = \frac{P}{A_o}$$

- On the contrary when changing area is taken into account, the ratio of load to actual area (A) is called 'true stress'.

$$\text{True stress} = \frac{P}{A}$$

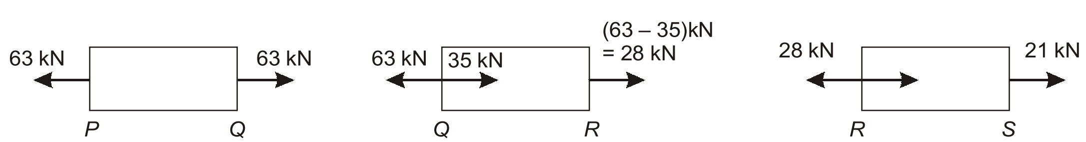
Example 1.2 A cross-section bar of area 700 mm² is subjected to an axial load as shown in the figure below, what is the value of stress (MPa) in the section QR?



(a) 30 (b) 40 (c) 50 (d) 60

[SSC JE : 2018]

***Solution:* (b)**



$P_{PQ} = 63$ kN (T) $P_{QR} = 28$ kN (T) $P_{RS} = 21$ kN (C)

$$\sigma_{QR} = \frac{P_{QR}}{A} = \frac{28000}{700} = 40 \text{ MPa}$$

1.4.2 Actual Curve Vs. Engg. Curve in Tension

NOTE

- The fracture strain depends on % carbon present in steel.
- With increase in % carbon, fracture strain reduces.
- With increase in carbon content, steel has higher yield stress and higher ultimate stress.

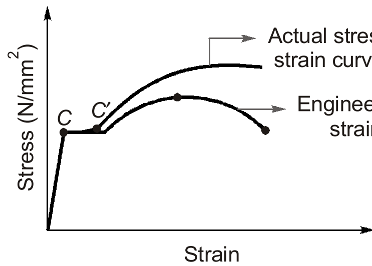


Figure 1.3

1.4.3 Compression Curve for Mild Steel

- In compression, engineering stress-strain curve lies above the actual stress-strain curve.
- In compression mild steel has yield stress σ_y = 263 N/mm², slightly greater than tension.
- Stress-strain curve for other grades of steel in tension.
- All the grades of steel have same.
- Among all steel grades, high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grade of steel.

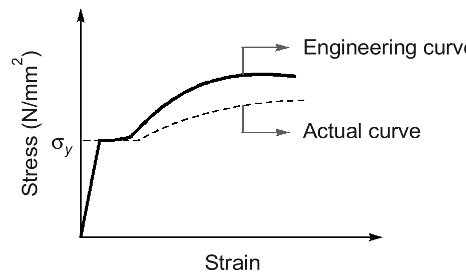


Figure 1.4

1.5 Properties of Metals

1.5.1 Ductility

- Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. Some of the examples are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze, monal metal etc.

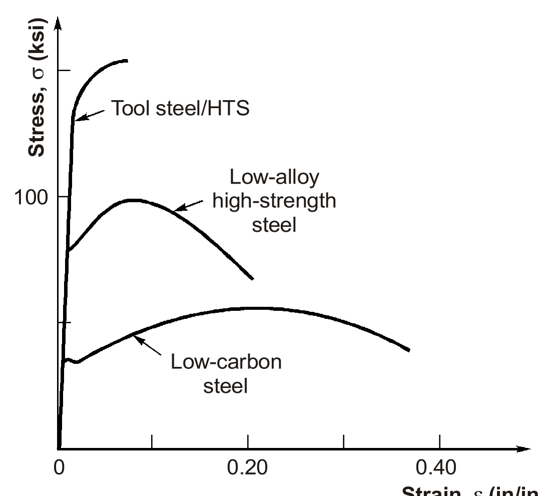


Figure 1.5

Example 1.3 The percentage reduction in area in case of cast iron when it is subjected to tensile test is of the order of

(a) 0% (b) 10% (c) 20% (d) 25%

[SSC JE: 2018]

Solution: **(a)**

The % reduction in area in case of cast iron when it is subjected to tensile test is of the order of 0%, because cast iron being brittle material does not undergo elongation.

1.5.2 Brittleness

- Brittleness is the lack of ductility i.e. material cannot be stretched. In brittle materials, failure takes place with a relatively smaller deformation. This property is undesirable. For brittle materials fracture point and ultimate points are same, and after proportional limit very small strain is seen. Some of the examples are cast iron, concrete and glass.

- To distinguish between these two types of materials, materials with strain less than 5% at fracture point are regarded as brittle and those having strains greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

1.5.3 Malleability

- The property by which a material can be uniformly extended in a direction without rupture. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

1.5.4 Toughness

- The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading.
- The **modulus of toughness** is measured as area under entire stress-strain curve and is the energy absorbed by material of the specimen per unit volume upto fracture stage.

$$\text{Modulus of toughness} = \left[\frac{\sigma_y + \sigma_u}{2}\right] \in_f$$

Where, σ_y = Yield tensile strength
σ_u = Ultimate tensile strength
$\in_f$ = Strain at fracture point

- The modulus of toughness will depend upon ultimate tensile strength and strain at failure (fracture strain). Hence the material which is very ductile will exhibit a higher modulus of toughness as the case with mild steel.
- The **modulus of resilience** is the maximum elastic energy per unit volume that can be absorbed without attaining plastic stage.

$$\text{Modulus of resilience } (u) = \frac{\sigma_y^2}{2E}$$

- **The modulus of resilience** depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.
- Higher toughness is a desirable property in materials used for gears, chains, crane hooks, freight car etc. Higher resilience is desirable in springs.

1.5.5 Hardness

- Hardness is defined as the resistance to indentation or scratching or surface abrasion.
- There are two methods of hardness measurement:
  1. Scratch hardness - commonly measured by Mohr's test.
  2. Indentation hardness (abrasion) measured by
- Brinell hardness method
- Rockwell hardness
- Vickers hardness
- Knoop hardness

It should be noted that ductile materials are tough and brittle materials are hard.

1.5.6 Fatigue

- It has been found that material behave differently under the static loading and dynamic loading.
- The behaviour of material under variable loads (dynamic loads) is referred to as **fatigue**. In recent past several failures of structures have been noted due to fatigue.
- Factors affecting fatigue are:
  1. Loading conditions
  2. Frequency of loading
  3. Corrosion
  4. Temperature
  5. Stress concentration

1.5.7 Creep and Stress Relaxation

- At any temperature, a material will progressively deform with the passage of time under constant loading, even if the stress is below yield point. This phenomenon is called creep. However, such deformation is negligibly small at lower temperature.
- At higher temperature, due to greater mobility of atoms, most of the materials loose their strength and elastic constants also get reduced. Hence, greater deformations at elevated temperature results even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
- The temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and is known as **homologous temperature**.
- If a wire of metal is stretched between two immovable supports, so that it has an initial tension stress σ_o. The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is a manifestation of creep, is called **stress relaxation**.

Do you know? For metallic minerals creep becomes an important consideration at half of the melting point temperature on absolute scale.

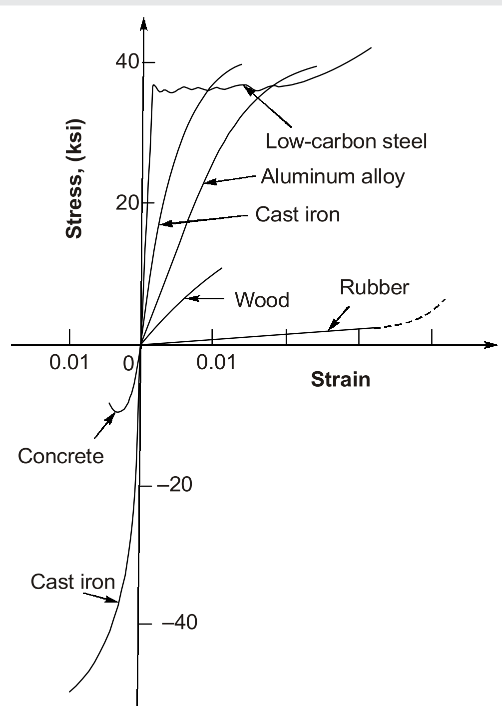


Figure 1.6 : Stress-strain diagram for different material

- Strain energy of a prismatic bar with varying section is given by

$$U = \frac{P^2}{2E}\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}+\ldots\frac{L_n}{A_n}\right]$$

- Strain energy of a prismatic bar hanging under its own weight is given by

$$U = \frac{\gamma^2 AL^3}{6E}$$

- Strain energy of a hanging elastic body due to more than one load cannot be found merely by adding the energies obtained from individual loads. This is because of the fact that strain in such a case is not a linear function but is a quadratic function of the loads.

Example 1.10 The strain energy per unit volume of a round bar under uni-axial tension with axial stress σ and modulus of elasticity *E* is

(a) $\frac{\sigma^2}{E}$ (b) $\frac{\sigma^2}{2E}$ (c) $\frac{\sigma^2}{3E}$ (d) $\frac{\sigma^2}{4E}$

[ESE : 2016]

Solution: **(b)**

$$\text{Strain energy/volume} = \frac{1}{2}\times\sigma\times E = \frac{1}{2}\times\sigma\times\frac{\sigma}{E} = \frac{\sigma^2}{2E}$$

Example 1.11 Total area under the stress-strain curve of a mild steel specimen tested up to failure under tension is a measure of its

(a) Breaking strength (b) Toughness (c) Hardness (d) Stiffness

[DMRC : 2015]

Solution: **(b)**

STUDENT'S ASSIGNMENT

Q.1 The ratio of Young's modulus to modulus of rigidity for a material having Poisson's ratio 0.2 is

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

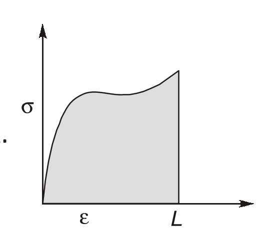
(c) $\frac{5}{14}$ (d) $\frac{14}{5}$

Q.2 A straight wire 15 m long is subjected to tensile stress of 2000 kgf/cm^2. Elastic modulus is 1.5×10^6 kgf/cm^2. Coefficient of linear expansion for the material is 16.66×10^{-6}/°F. The temperature change (in °F) to produce the same elongation as due to the 2000 kgf/cm^2 tensile stress in the material is:

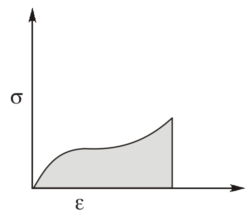
(a) 40 (b) 80

(c) 120 (d) 160

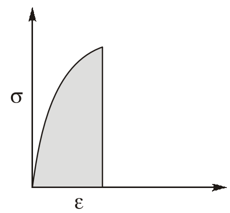
Q.3 Match **List-I** with **List-II** and select the correct answer using the codes given below the lists:

List-I	List-II
1. Soft & Weak	A. 

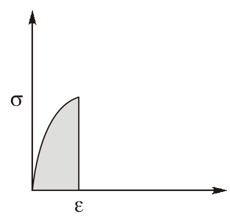
2. Hard brittle B.



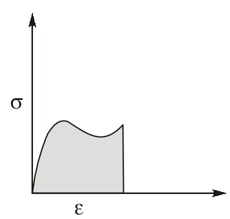
3. Hard strong C.



4. Soft tough D.



5. Hard tough E.

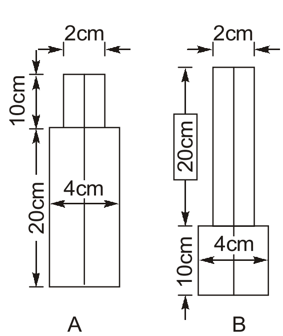


Codes:

	1	2	3	4	5
(a)	E	D	C	B	A
(b)	A	B	C	D	E
(c)	E	C	B	D	A
(d)	A	D	C	B	E

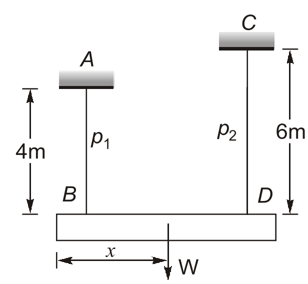
Q.4 Two similar round bars *A* and *B* are each 30 cm long as shown in the given figure. The ratio of strain energies stored by the bars *A* and *B*, $\dfrac{U_B}{U_A}$ is

(Assume both bars to be axially loaded equally)



(a) 3/2 (b) 1.0
(c) 5/8 (d) 2/3

Q.5 A gradually applied load *W* is suspended by wire ropes *AB* and *CD* as shown in the figure. The wires *AB* and *CD*, made of the same material and of the same cross-section are connected to a rigid block from which the load *W* is suspended in such a way that both the ropes stretch by the same amount. If the stress in *AB* and *CD* are p_1 and p_2 respectively, then the ratio $\dfrac{p_1}{p_2}$ will be



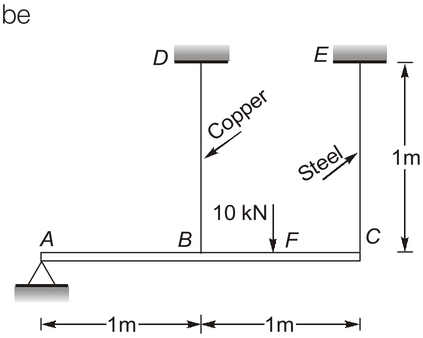
(a) 3/2 (b) 2/3
(c) 9/4 (d) 4/9

Q.6 *ABC* is a rigid bar. It is hinged at *A* and suspended at *B* and *C* by two wires, *BD* and *CE* made of copper and steel respectively, as shown in the given figure. The bar carries a load of 10 kN at *F*, midway between *B* and *C*.

Given that

$A_c = 4\text{cm}^2$, $A_s = 2\text{cm}^2$, $E_c = 1 \times 10^5$ N/mm^2,
$E_s = 2 \times 10^5$ N/mm^2

Subscript c and s stands for copper and steel. If the extensions in the steel and copper wires are Δ_s and Δ_c respectively, the ratio $\dfrac{\Delta_s}{\Delta_c}$ would be



the axial stress induced in the rod due to this rise in temperature is (for steel E = 200 GPa, $\alpha = 11.5 \times 10^{-6}$/°C).
(a) 46 MPa tension
(b) 46 MPa compression
(c) 23 MPa tension
(d) 23 MPa compression

[DRDO : 2009]

Q.16 If a material expands freely due to heating it will develop
(a) thermal stress (b) tensile stress
(c) no stress (d) bending

[ISRO : 2007]

Q.17 If Poisson's ratio for a material is 0.5, then the elastic modulus for the material is
(a) 3 times its shear modulus
(b) 4 times its shear modulus
(c) Equal to its shear modulus
(d) Indeterminate

[ISRO : 2006]

Q.18 The number of independent elastic constraints required to express the stress-strain relationship for a linearly elastic isotropic material is
(a) One (b) Two
(c) Three (d) Four

[ESE : 1998]

Q.19 What is the strain energy stored in a body of volume V with stress σ due to gradually applied load?

(a) $\frac{\sigma E}{V}$ (b) $\frac{\sigma E^2}{V}$

(c) $\frac{\sigma V^2}{E}$ (d) $\frac{\sigma^2 V}{2E}$

[ESE : 2006]

ANSWER KEY // STUDENT'S ASSIGNMENT

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)
6. (c)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (b)
11. (b)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (b)
16. (c)	**17.** (a)	**18.** (b)	**19.** (d)	

HINTS & SOLUTIONS // STUDENT'S ASSIGNMENT

1. (d)

$$G = \frac{E}{2(1+\mu)}$$

$$\frac{E}{G} = 2 \times 1.2 = 2.4 = \frac{12}{5}$$

2. (b)

$$\Delta = \frac{\sigma L}{E} = L\alpha\Delta T$$

$$\therefore \quad \Delta T = \frac{\sigma}{E\alpha}$$

$$= \frac{2000}{1.5 \times 10^6 \times 16.66 \times 10^{-6}}$$

$$= 80.03° \text{ F}$$

3. (a)

1-E, 2-D, 3-C, 4-B, 5-A

Toughness is measured by area under stress-strain curve. Brittleness is measured by strain at failure.

Higher the yield point, the metal will be harder.

Strong/weak can be measured by ultimate tensile strength or peak of stress-strain curve. Based on these concepts the answer can be easily found.

4. (a)

$$U = \frac{P^2L}{2AE} \text{ For axially loaded bar.}$$

$$U_A = \frac{P^2L_1}{2A_1E} + \frac{P^2L_2}{2A_2E};$$

$$U_B = \frac{P^2L_2}{2A_1E} + \frac{P^2L_1}{2A_2E}$$

5. (a)

$$\Delta = \frac{p_1 l_1}{EA} = \frac{p_2 l_2}{EA}$$

$$\Rightarrow \quad \frac{p_1}{p_2} = \frac{l_2}{l_1} = \frac{6}{4} = \frac{3}{2}$$